### Written Exam at the Department of Economics winter 2016-17

#### **Corporate Finance and Incentives**

Final Exam/ Elective Course/ Master's Course

February 15, 2017

(3-hour closed book exam – access to Excel)

Please note that the language used in your exam paper must correspond to the language for which you registered during exam registration.

This exam question consists of 3 pages in total

*NB:* If you fall ill during the actual examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

The exam consists of 4 problems. All problems must be solved. The approximate weight in the final grade of each problem is stated. A problem can consist of different sub-questions that do not necessarily have equal weight. Please provide intermediate calculations.

### Problem 1 (Asset Pricing 20%)

Four assets i = 1, ..., 4 are traded today at prices  $\varphi^T = (9, 6, 6, 25)$ . They deliver cash flow at a future date when the economy will be in one of four possible states j = 1, ..., 4. Cash flows are listed in matrix V, where the *i*'th column represents asset *i*, while rows correspond to states:

$$V = \begin{bmatrix} 10 & 1 & 5 & 24 \\ 5 & 1 & 7 & 53 \\ 7 & 1 & 4 & 31 \\ 14 & 20 & 9 & 2 \end{bmatrix}$$

1) Find a vector d that solves  $\varphi^T = d^T V$ .

2) Do the prices of the four assets permit arbitrage?

3) Find a risk-free interest rate r and a probability vector p such that the price for each asset i satisfies

$$\varphi_i = \frac{1}{1+r} \sum_{j=1}^4 p_j V_{ji}.$$

4) Now, add an asset with cash flows  $V_5^T = (7, 4, 6, 1)$ . The original four assets keep their prices from before. What market price do you expect to see on this fifth asset, and why?

# Problem 2 (Debt 30%)

Consider a firm that initially has no debt. The firm has an asset which will pay a safe profit of 105 million Kroner one year from today. No profits arrive after that date.

Capital markets are perfect, with a risk-free interest rate set to r = 5%. The corporate tax rate is  $\tau_c = 30\%$ . The personal tax rate on interest income is  $\tau_i = 40\%$ . The personal tax rate on equity income is  $\tau_e = 20\%$ .

In the initial situation, next year the firm pays corporate taxes on all 105 million Kroner. The remaining amount is returned to shareholders, who pay personal tax on this income.

1) Compute the present values of equity and of the total tax bill.

The firm has decided to borrow some money today, by promising to pay the amount 50 million Kroner to creditors next year.

2) Explain why the present value D < 50 of debt should satisfy

$$D = \frac{D + (1 - \tau_i) (50 - D)}{1 + r},$$

and compute D as well as the amount of interest on this debt.

The firm pays out D to shareholders today, taxed at their personal income tax rate.

Next year, the firm can deduct the interest payment from its corporate tax base.

3) How much remains next year after repaying creditors (with interest) and paying corporate tax? Compute also the personal income tax paid by equity holders next year.

4) Compute the present value of the total tax bill, including today's shareholder tax.

5) Do shareholders benefit from this releveraging of the firm?

# Problem 3 (Real Options 25%)

A firm faces a dynamic investment problem. It has developed a new gadget, but needs to invest into marketing before it can start to earn any profits from sales.

Today, the firm can invest either 0 or the amount  $I_0 > 0$ .

One month later, it will be revealed whether demand for the gadget is high or low. The market attaches probability 40% to the high demand state. The risk-free interest rate is 0.

Next month, after seeing the demand state, the firm can invest either 0 or  $I_1 > 0$ .

The numbers  $I_0$  and  $I_1$  are exogenously given.

- If both investments are strictly positive, the firm will get an income stream that has present value 200 in the high demand state, and 50 in the low demand state.
- If the investment is  $I_0 > 0$  today and 0 next month, the firm will get income of present value 100 in the high demand state, and 50 in the low demand state.
- If the investment is 0 today and  $I_1 > 0$ , the firm will get income of present value 40 in the high demand state, and 40 in the low demand state.
- If the investment is 0 at both times, the firm gets no income.

1) Suppose the firm has already invested  $I_0 > 0$ , and learned the demand state. What is the optimal thing to do next month, in each state, as a function of  $I_1$ ?

2) Suppose the firm invested 0 today, and has learned the demand state. What is the optimal thing to do next month, in each state, as a function of  $I_1$ ?

3) As a function of  $I_0$  and  $I_1$ , what is the optimal thing to do today? You may find it useful to consider three cases, where  $I_1$  is either low, medium, or high.

# Problem 4 (Various Themes 25%)

1) The model for the Arbitrage Pricing Theory (APT) assumes that the return on asset i can be expressed as  $r_i = \alpha_i + \sum_{k=1}^{K} \beta_{ik} F_k + \varepsilon_i$ . Explain the ingredients in this equation. What does the APT predict that the risk premium of asset i must satisfy?

2) Give a short derivation of the put-call parity for European options that have identical exercise prices on a common expiration date.

3) Suppose Donald Trump allows U.S. corporations to bring home profits currently parked as cash in foreign subsidiaries, without being subject to U.S. taxation. It is the hope that this cash will increase the macro-level of investments in the U.S.. Discuss whether it is reasonable to hope for such repatriation of profits, and whether the investment effect should be expected.